

# An Introduction to Statistics, Measurements and Error Theory



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"There are three kinds of  
lies:  
lies,  
*damned lies,*  
and statistics."

--Samuel Clemens

Most people know that statistics can often be presented in a way that supports a particular view or contention. For example, one can cite the *median income* or the *average income*, but two statistical terms do not mean the same thing mathematically. The median is the "middle" value in a set of data that is organized from the smallest value to the largest value. This means that there are equal numbers of people earning less and more than the median income value. But the median value can be very different than the mean or average income, depending on the distribution of incomes in the data set—as the following example shows:

XYZ Surveying is recruiting for Field Technicians. In their recruiting literature, they say

*... "at XYZ surveying, we offer competitive salaries. Our average rate of compensation is \$45382.00"*

Being a savvy person, you go to the LSAW website and look up the average salary for field technicians in Podunk County, where XYZ is located. There you find that, on average, field technicians earn only \$35160.00. Wow, maybe you can get more by working for XYZ!

## What the statistics mean depends on the data...

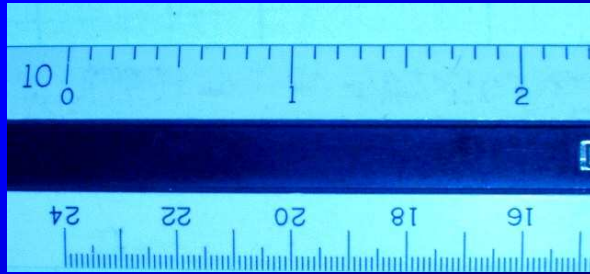
	Yearly	Hourly
Principal Surveyor	\$ 82,500.00	\$ 39.66
Project Surveyor	\$ 64,000.00	\$ 30.77
Senior Technician	\$ 44,720.00	\$ 21.50
Field Technician	\$ 35,160.00	\$ 16.90
Field Technician	\$ 35,160.00	\$ 16.90
Field Technician	\$ 35,160.00	\$ 16.90
Field Technician	\$ 35,160.00	\$ 16.90
Clerical Staff	\$ 31,200.00	\$ 15.00
<b>Sum =</b>	<b>\$ 363,060.00</b>	
<b>Average =</b>	<b>\$ 45,382.50</b>	
<b>Median =</b>	<b>\$ 35,160.00</b>	

In looking at this dataset, we can see that the recruiter chose to emphasize the Average Income, instead of the Median Income. Which value, Average or Median, gives the more accurate picture of what a field technician can expect to make at XYZ Surveying?

Obviously, this is a very simplistic example of how statistical information can be used to “prove” a point. I bring this to your attention mainly to remind you that one should ALWAYS be thinking about the dataset from which statistical information is derived. What are some of the problems with this data set?

# Measuring and Counting

- Surveyors measure, laypeople count
- *This is why laypeople don't always understand us, or what we do*
- Consider an Engineer's Scale:



The reason clients and the public say things like, "Why is it you surveyors can't agree on the length of this line?!", is because they think in terms of *counting* whereas we surveyors *measure*.

As expert measurers, we fully understand that our measurements may not agree exactly with values that other surveyors report. Differences may be caused by different equipment, procedures or other factors. For instance, you might be able to measure a particular distance directly, while an earlier survey might have required a multi-station traverse (also known as an *indirect measurement*) to measure the same line.

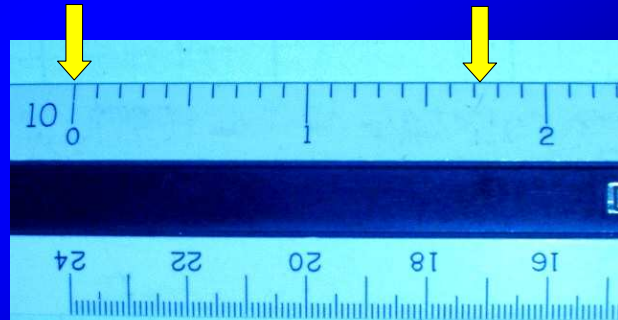
But perhaps more importantly, counting involves determination of discrete, indivisible quantities: The number of people in this room is a whole number. One can simply count heads, and presuming that one can count properly, the count will be without error, and another person who repeats the count will come up with the same answer.

If however we were to bring two bathroom scales into the room and use them to find the total weight of all the participants of this seminar, I daresay we'd get similar, but different answers from each scale.

Counts and measurements differ in another way too....Let's take a look at the process of *measuring*....

## Taking a measurement

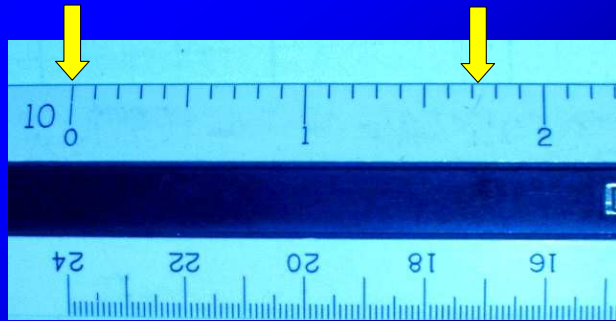
- The first arrow falls at the zero mark....
- The second arrow falls between 1 and 2, so the first *certain* digit in our measurement is "1"



Measuring is a *process*...

## Taking a measurement

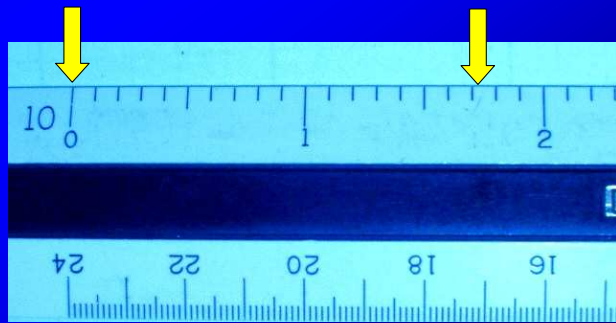
- The second arrow falls between 1.7 and 1.8, so the second *certain* digit in our measurement is a "7"



Measuring is a process...

## Measuring involves *estimating...*

- The third digit in our measurement must be estimated, so its value is *uncertain...is it a "2" or a "3"?*



Measuring is a *process* that, unlike counting, involves *estimation*. No matter the precision of the measuring instrument, sooner or later an estimate must be made...

## ...and estimates result in small (but not insignificant) errors

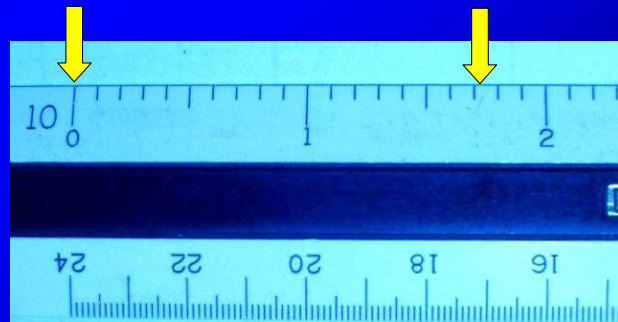
- Is the final measurement 1.72 or 1.73?
- Different people are likely to have different opinions....



...and estimates result in errors. Counting does not involve estimating, so counted quantities do not have estimation errors associated with them, but every measurement has some error.

## 1.72 or 1.73?

- The difference between the measurement and "Truth" is error



Truth is hard to come by. Our measurements are *estimates of the truth*.

# Types of Errors

- Systematic Errors
- Random Errors
- Blunders

**All measurements contain error. Errors come from different sources**

**Systematic Errors:** Errors that can be computed and subsequently eliminated from a system of measurements. Example: Error in a chained distance caused by thermal expansion of the steel tape.

**Random Errors:** Errors that naturally occur in a system of measurements. Random errors are the only class of errors that remain in measurement system after blunders and systematic errors have been eliminated. Random errors' magnitudes and algebraic signs cannot be predicted with certainty. Nevertheless, their effects can be estimated. Random errors in a measurement system do not accumulate as systematic errors do, instead they *propagate* according to the square root of the sum of the squares.

**Blunder:** A screw up. A mistake. An erroneous value resulting from a screw up or mistake. These have no place in a system of measurements as they produce an unwanted and unwarranted influence on the mean and other computed statistics, such as the Standard Deviation and Standard Error.

## Errors and Blunders

- In any measurement system, errors are a natural and expected occurrence
- Errors and blunders are not the same thing!
- Errors result from *imperfections* in the measurement system
- Blunders come from incompetence, carelessness, lack of training, poor procedures, broken equipment

No tool used for surveying measurements is perfect, therefore every measurement contains some level of uncertainty or *error*, no matter how small. Steel tapes sag and expand or contract with changes in temperature. Rag tapes stretch. No two EDMs or prisms are perfectly identical and so return slightly different measurements. Operators make imprecise readings and don't get set-up perfectly at each station. Even the most sophisticated theodolite is imperfectly made.

*Errors* cannot be avoided and are present in every measurement, but they are different from *blunders*. *Blunders* are foul-ups and must be eliminated from your measurements. Setting up on the *wrong point*, failure to set up the equipment plumb over a point, transposing numbers in the field notes, or sighting the wrong backsight are examples of blunders.

# Elimination of Errors and Blunders

- Goal: *prevent* blunders
  - field procedures are designed to do this
- Goal: *eliminate, or reduce the effects of errors on measured values.*



## Prevention of Blunders

Field procedures such as direct and reverse angle observations or two independent distance measurements in a “fly-tie” are designed to prevent and trap blunders. In the case of a property corner stake-out, measuring angles from two different backsights ensures that the coordinate for the tied point is correctly computed!

**Dabob School project:** A fly-tie was performed from an incorrectly marked traverse station, resulting in a bad coordinate on a  $\frac{1}{4}$  corner. Although correct field procedures were followed (double backsights were used and the distance to the monument was double measured), the office staff failed to compute two sets of coordinates from the two sets of field data, so a bad coordinate was developed for the  $\frac{1}{4}$  corner. Result: the subdivision of two adjoining sections was done improperly and the entire project had to be redone.

## Reduction of Errors

Errors are controlled by keeping the equipment in good adjustment and/or by using observing procedures that cause error sources to cancel out. Observing forward and reverse positions when measuring angles or when extending a line causes many instrument errors to cancel out.

## Behavior of Systematic Errors

- Magnitude is predictable
- Algebraic sign is predictable
- Caused by-
  - environmental conditions
  - maladjustment
  - imperfect manufacture
  - bias of observer



### **Systematic error:**

Errors that can be computed and subsequently eliminated from a system of measurements.  
Example: Error in a chained distance caused by thermal expansion of the steel tape.

## Sources of Systematic Errors

- Natural
- Instrumental
- Personal

**Natural Errors:** Errors in a measurement that come from environmental sources, for example, changes in temperature, and barometric pressure affect measurements collected with an EDM. Natural errors can be random or systematic in nature. The error in the EDM example given here can be compensated for if the temperature and pressure are observed when the measurements are taken. This is an example of a systematic error. Heat waves can make a distant target “dance around” in the telescope and will probably result in larger random errors in pointing the instrument than would otherwise be the case.

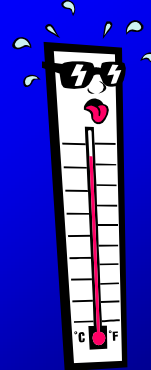
**Instrumental errors:** Errors in a measurement system arising from imperfections in the tools or instruments used to obtain the observations.

**Personal errors** in a measurement system are caused by the physical limitations or biases of the observer. They are mostly random in nature, but an observer’s *bias* could manifest itself as a tendency to always “read high,” thus *skewing* the data set.



## Sources of Systematic Errors

- Natural Errors
  - errors induced in a measurement system by environmental factors
  - temperature
  - barometric pressure



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## Sources of Systematic Errors

- Instrumental Errors
  - errors induced in a measurement system by imperfectly made tools
  - plate eccentricity
  - axis mis-alignment
  - EDM phase center
  - imperfect graduations



**Instrumental errors:** Errors in a measurement system arising from imperfections in the tools or instruments used to obtain the observations.

## Sources of Systematic Errors

- Personal Errors
  - Errors induced in a measurement system by observers
  - observers' tendency to take "biased" readings



We are only human. Individuals might have a tendency to read a little high or a little low, or maybe a little to the left or right.

## Eliminating Systematic Errors

- Effects from a single source *accumulate*
- Must be eliminated from data either mathematically or by use of proper procedures

When you apply a PPM correction to an observed EDM distance to compensate for temperature and barometric conditions, you are using mathematics to eliminate (or reduce to a negligible level) a systematic error source.

When you take direct and reversed observations with a theodolite, you are using a procedure that can cancel out several instrumental errors associated with angular measurements, namely axis alignment problems.

# Eliminating Systematic Errors

- Examples:
  - Applying temperature and barometric pressure corrections to distances measured with a EDM
  - Applying a grid scale factor to measured ground distances for WCS jobs.
  - "Direct" and "reverse" angle readings eliminates many instrumental errors associated with angle measurements

## Eliminating Systematic Errors

- Keep those instruments in good adjustment, and keep a log!
- Calibrate EDM at least yearly
- Measure and record environmental conditions
- Follow measurement procedures

It is good practice to initiate a regularly scheduled instrument maintenance program. In addition to daily checks and adjustments, consider performing regular baseline, collimation and optical plummet tests. The results of these tests should be documented in a special field book kept only for that purpose. That way you can spot any developing trends.

## Behavior of Random Errors

- Magnitude impossible to predict
- Algebraic sign impossible to predict
- Caused by-
  - limitations of observers
  - limitations in design of instrument
  - maladjustment
- Effects cannot be eliminated, only minimized

**Random errors:** Errors that naturally occur in a system of measurements. Random errors are the only class of errors that remain in measurement system after blunders and systematic errors have been eliminated. Random errors' magnitudes and algebraic signs cannot be predicted with certainty. Nevertheless, their effects can be estimated. Random errors in a measurement system do not accumulate as systematic errors do, instead they *propagate* according to the square root of the sum of the squares.

## Behavior of Random Errors

- Effects *tend* to compensate
- Obey laws of probability-
  - small random errors more frequent
  - large random errors less frequent
  - fall within a predictable range
- Effects distributed according to the laws of probability
  - Propagate by the "square root of the sum of the squares"

Since the algebraic signs and magnitudes of random errors vary, they never completely cancel out. The odds are such that small-magnitude random errors will occur more frequently than large ones. Random errors do not accumulate as systematic errors do, instead they *propagate according to the square root of the sum of the squares*.

## Behavior of Random Errors

- Effects distributed according to the laws of probability
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*Example:* A distance of 989.45 feet is measured with a 100 foot tape in 10 segments. If each segment contains a random error of +/- 0.012 feet, what is the total probable error in the measurement?

Since we are analyzing the effects of *random errors*, the errors in each segment do not accumulate arithmetically; instead they *propagate according to the square root of the sum of the squares*. Since the errors can be positive or negative, they will *tend* to cancel out, so the total probable error in the measurement will be less than the arithmetic sum of the errors of the individual measurements...

$$Error = \sqrt{(0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2 + 0.012^2)}$$

Or

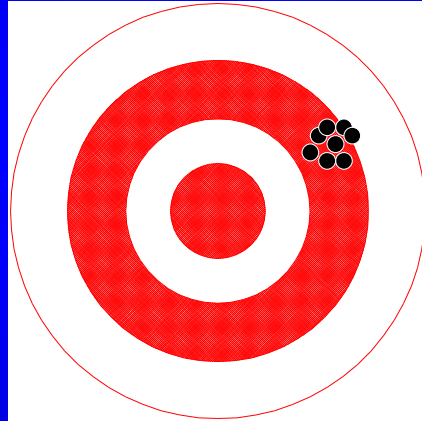
$$Error = \sqrt{10 \times (0.012^2)} = 0.038$$

## Minimizing Random Errors

- Know capabilities of instruments and their operators
- Keep instruments properly adjusted
- Example: optical plummet maladjustment induces a random instrument centering error

# Precision

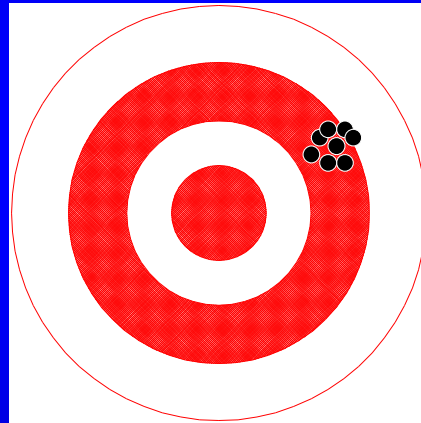
- Deals with degree of refinement in measurement
- Ability to reliably repeat a measurement
- Not always well related to "true value," esp. if systematic errors are present



**Precision:** The degree of refinement of a measurement or set of measurements.

# Precision

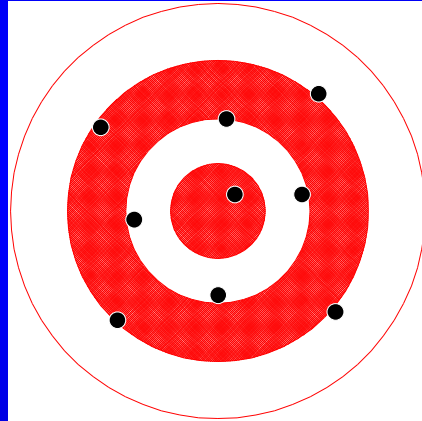
- Low "scatter" indicates high precision...
- ...but consistent deviation from center indicates low accuracy
- Systematic errors responsible for poor accuracy (bad backsight reading?)



**Scatter:** The degree to which a set of data is dispersed with respect to its mean. High scatter results in larger Standard Deviations, and implies less precision.

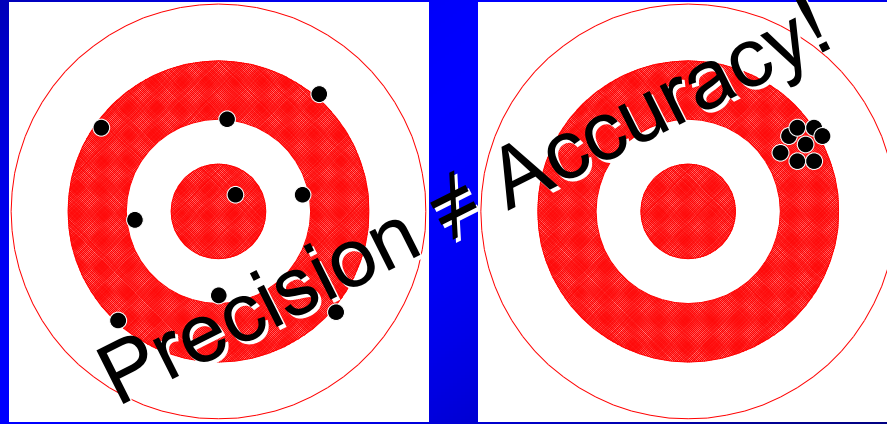
## Accuracy

- Accurate, but not precise
- Wide, uniform "scatter" indicates a lack of precision
- Shots centered in target, shows some level of accuracy
- What factors are responsible for poor precision?



**Accuracy:** The degree to which a measured quantity is related to the "true value." The closer a measurement (or the mean of a set of measurements) is to the true value, the greater the accuracy of the measurement.

If you averaged the shots, in each case, which would be closer to "truth"?

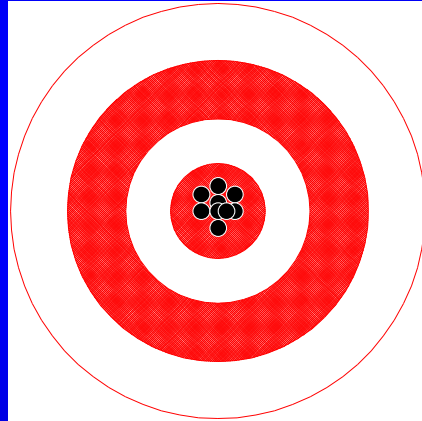


**Accuracy:** The degree to which a measured quantity is related to the "true value." The closer a measurement (or the mean of a set of measurements) is to the true value, the greater the accuracy of the measurement.

# Accuracy

- This is what we *want*:

high precision, high  
accuracy  
measurements



To get high precision and high accuracy you must:

- Eliminate all blunders from all measurements
- Eliminate or compensate for all systematic errors
- Reduce the magnitudes of random errors to their absolute minimums

To do this...

- Devise and follow procedures that are designed to prevent or catch blunders
- Apply all correction factors, such as scale factors, prism offsets, temperature and barometric pressure corrections, etc.
- Keep all equipment in proper adjustment and calibrated

# Theory of Measurements

- No measurement is exact
- Every measurement contains error
- The true value = measurement - error
  - the exact value of the error is unknown
  - therefore, the true value can never be determined
- Most likely value = average or mean of many observations

**True value:** A measurement minus random error. Since all measurements contain *some* random error, it is impossible to determine a true value *exactly*, though it is possible to estimate the *how close* the mean of a series of measurement is to the true value.

**Mean:** The average of a series of measurements. The *most likely value* for a measurement that consists of multiple observations.

## The Mean: A Measure of Central Tendency

- The true value of a measurement cannot be determined
- The mean is our *best estimate* of a measured quantity
- The mean *approaches* the true value as number of observations approaches infinity

**Mean:** The average of a series of measurements. The *most likely value* for a measurement that consists of multiple observations. To get a better estimate of a measured quantity, *take more observations!*

## Other Measures of Central Tendency

- Median - The "middle value" in a set of observations
- Mode - The most frequently occurring value in a set of observations

**Median:** In a data set, the median is the "middle value." In a set of data, there are an equal number of values that are greater than and less than the median.

Data: 4.0, 5.0, 5.0, 5.2, 6.3, 7.9,  
9.9

- Mean = 6.2
- Median = 5.2
- Mode = 5.0

## Questions We Should Ask About Data:

- How many observations were made?
- How variable is the data?
- How close is the mean to the true value?

All data should be subjected to analysis. Don't take it for granted.

## Range

- One measure of variability is the *range* of a set of data
- Range = difference between largest and smallest values in the set of observations

**Range:** The difference between the smallest and the largest values in a set of data.

Data: 4.0, 5.0, 5.0, 5.2, 6.3, 7.9,  
9.9

• Range =  $9.9 - 4.0 = 5.9$

## Residuals

- One way to look at the quality of the mean is to compare it to each observation
- The difference between an observation and the mean is known as a *residual*, and is identified with the symbol " $v$ "
- $v = (\text{an individual measurement}) - (\text{the mean})$

**Residual:** An observation minus the mean of the set of observations. Residuals are used for computing Standard Deviation and Standard Error.

Data: 4.0, 5.0, 5.0, 5.2, 6.3, 7.9,  
9.9

Obs. #	value	-mean	v
1	4.0	-6.2	-2.2
2	5.0	-6.2	-1.2
3	5.0	-6.2	-1.2
4	5.2	-6.2	-1.0
5	6.3	-6.2	0.1
6	7.9	-6.2	1.7
7	9.9	-6.2	3.7

v = value-mean

# Mean Deviation

- Average of all residuals in the set
- Better than range...
- ...but it says nothing about how the data is dispersed

Obs. #	value	-mean	v
1	4.0	-6.2	-2.2
2	5.0	-6.2	-1.2
3	5.0	-6.2	-1.2
4	5.2	-6.2	-1.0
5	6.3	-6.2	0.1
6	7.9	-6.2	1.7
7	9.9	-6.2	3.7

$$\text{MeanDeviation} = \frac{\sum |v|}{n} = \frac{11.1}{7} = 1.6$$

**Mean deviation:** The mean of the absolute values of the residuals in a set of data. Does not address *scatter*: two data sets can have the same mean deviation, but vastly different scatter.

# Standard Deviation

- ...is a statement about the *precision* of the mean-
  - and of the *variability* of the set of data from which the mean is derived
  - the larger the standard deviation, the more variability (scatter) in the data
- ...can be used to predict the *range* in which any *future* single measurement will fall, given the same circumstances
- Also known as the *root mean square error*

**Standard Deviation:** An expression that describes the precision of the mean with respect to the data set from which it is derived. Small Standard Deviations imply a data set with low scatter and hence, high precision.

# Standard Deviation

- = the square root of (the sum of the squared residuals) divided by (the number of observations minus 1)

$$\sigma_s = \sqrt{\frac{\sum v^2}{n-1}}$$

The formula shown here is used for computing the Standard Deviation of a sample.

**Sample:** A subset of a *population*. In most instances it is not possible to measure an entire population, because some populations are infinite in nature. For the purposes of statistical analysis, a *sample* can often suffice.

**Population:** The entire set of all possible measurements in a data set. Some populations are finite (the set of bushel baskets of tomatoes harvested from my garden), others are not (the set of possible distance measurements from my SE to NE property corners (an infinite set limited by only by cost and patience—I have sons, so they *could* continue measuring long after my death...and *they* could have offspring, and on and on and on....)).

In computing the Standard Deviation of a population,  $n$  is used in the formula instead of  $n-1$ .

When using a calculator's or a spreadsheet's built-in statistics functions to calculate Standard Deviation, you should verify which formula is being used.

# Computing Standard Deviation

- Arrange observations in a table
- Find mean
- Find residuals
- Find squares of residuals, and sum them
- Apply formula

Obs #	value	-mean	v	v squared
1	4.0	-6.1857	-2.1857	4.7773
2	5.0	-6.1857	-1.1857	1.4059
3	5.0	-6.1857	-1.1857	1.4059
4	5.2	-6.1857	-0.9857	0.9716
5	6.3	-6.1857	0.1143	0.0131
6	7.9	-6.1857	1.7143	2.9388
7	9.9	-6.1857	3.7143	13.7959
Mean =	6.1857		Sum of Squared Residuals=	25.3086

$$\sigma_s = \sqrt{\frac{25.3086}{7-1}} = 2.0$$

The result in this case tells us that about 70% of the observations are within +/- 2.0 of the mean. This is a small sample...to small to be statistically significant, nevertheless, 5 of 7 values *do* fall between 4.2 and 8.2 – which is about 70% of the data.

# Computing Standard Deviation

- Given this data, a standard deviation of 2.0 means that the majority of the data (about 70%) falls within +/- 2.0 of the mean of 6.2

Obs #	value	-mean	v	v squared
1	4.0	-6.1857	-2.1857	4.7773
2	5.0	-6.1857	-1.1857	1.4059
3	5.0	-6.1857	-1.1857	1.4059
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6	7.9	-6.1857	1.7143	2.9388
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Mean =	6.1857		Sum of Squared Residuals=	25.3086

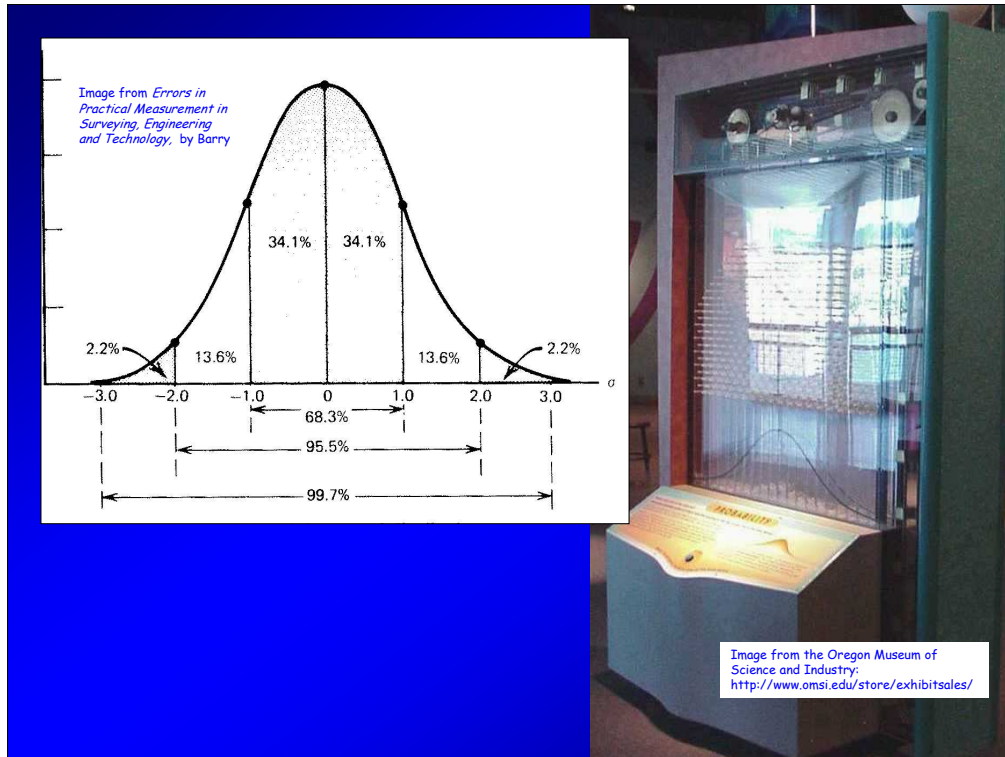
$$\sigma_s = \sqrt{\frac{25.3086}{7-1}} = 2.0$$

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## Standard Deviation at 1 Sigma

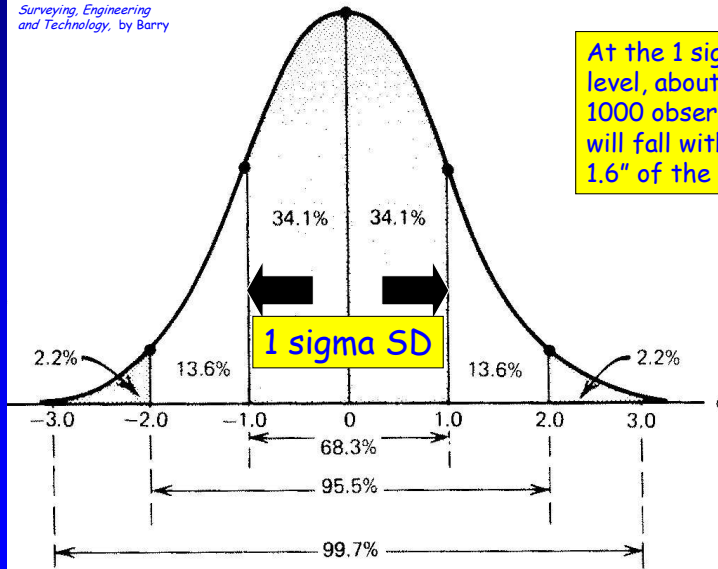
- The mean of a set of 1000 observations is 28" and the 1 sigma standard deviation is +/-1.6"
- This means:
  - 68.3% (683 of 1000) of the observations are within +/- 1.6" of the mean (from 26.4" to 29.6")
  - there is a 68.3% chance that a single, future observation will fall within this range.

**One Sigma Error:** Plus or minus 1 standard deviation. This value comprises 68.3% of the observations in a data set. The implication is that if an additional observation were taken, there is a 68.3% chance that it will fall within 1 standard deviation of the previously determined mean. It is computed by finding the square root of the sum of the squared residuals divided by the one less than the number of observations in the set.



**One Sigma Error:** Plus or minus 1 standard deviation. This value comprises 68.3% of the observations in a data set. The implication is that if an additional observation were taken, there is a 68.3% chance that it will fall within 1 standard deviation of the previously determined mean. It is computed by finding the square root of the sum of the squared residuals divided by the one less than the number of observations in the set.

Image from *Errors in Practical Measurement in Surveying, Engineering and Technology*, by Barry



At the 1 sigma level, about 683 of 1000 observations will fall with +/- 1.6" of the mean

**Fig 5.6** Characteristics of the normal distribution curve.

## Standard Deviation at 2 Sigma

- 2 sigma standard deviation =  $2 \times 1$  sigma standard deviation
- This means:
  - 95.5% (955 of 1000) of the observations are within +/- 3.2" of the mean (from 24.8" to 31.2")
  - there is a 95.5% chance that a single, future observation will fall within this range

**Two Sigma Error:** Plus or minus 2 Standard Deviations. This value comprises 95.5% of the observations in a data set. The implication is that if an additional observation were taken, there is a 95.5% chance that it will fall within 2 Standard Deviations of the previously determined mean. See also *one sigma error*

Image from *Errors in Practical Measurement in Surveying, Engineering and Technology*, by Barry

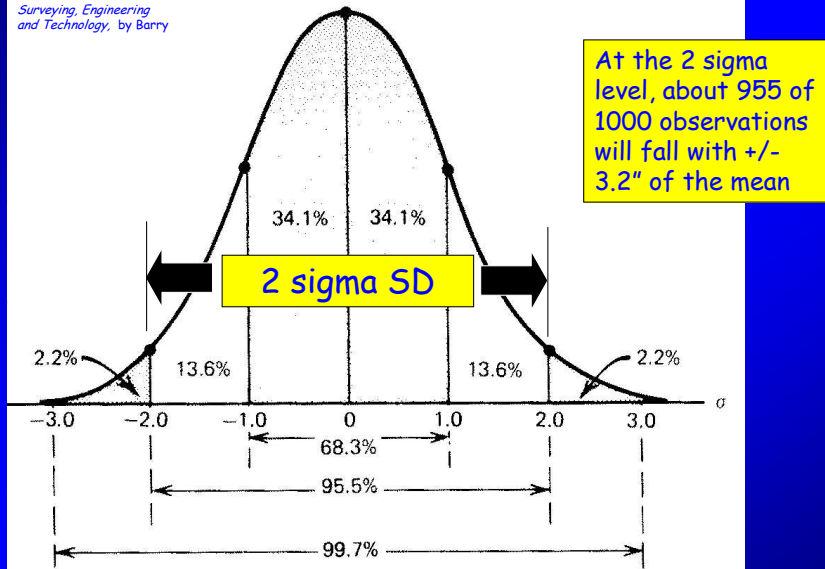


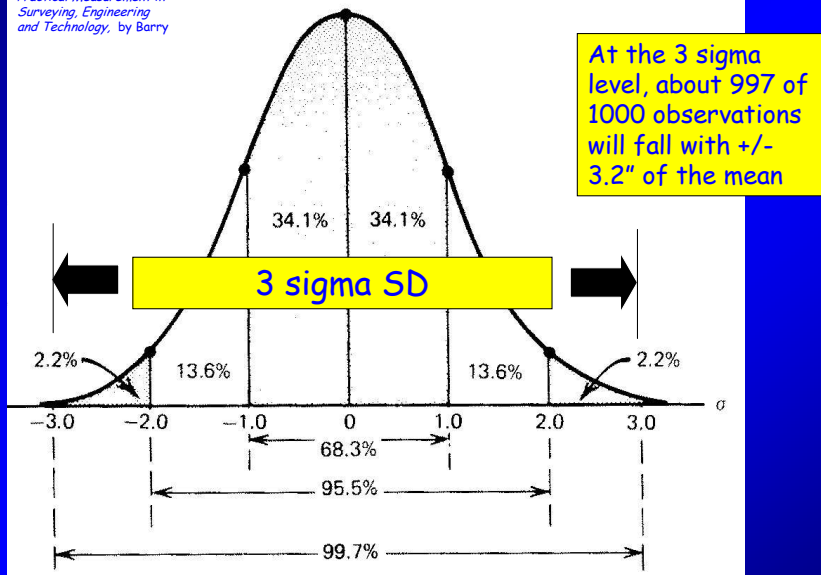
Fig 5.6 Characteristics of the normal distribution curve.

## Standard Deviation at 3 Sigma

- 3 sigma standard deviation =  $3 \times 1$  sigma standard deviation
- This means:
  - 99.7% (997 of 1000) of the observations are within +/- 4.8" of the mean (from 23.2" to 32.8")
  - there is a 99.7% chance that a single future observation will fall within this range

**Three Sigma Error:** Plus or minus 3 Standard Deviations. This value comprises 99.7% of the observations in a data set. The implication is that if an additional observation were taken, there is a 99.7% chance that it will fall within 3 Standard Deviations of the previously determined mean. See also *one sigma error*

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**Fig 5.6** Characteristics of the normal distribution curve.

## Standard Deviation

- Is a statement of probability regarding relationship between a mean and its data
- Is valid if and only if:
  - test conditions and field conditions are the same
  - no systematic errors are present
  - no mistakes are present

You can't evaluate your data unless it is free of blunders and systematic errors.

## Standard Error

- ...is a statement regarding the uncertainty of, and thus the *accuracy* of, the mean
- Is valid if and only if:
  - test conditions and field conditions are the same
  - no systematic errors are present
  - no mistakes are present

**Standard Error:** An expression that describes the relationship of the mean of a set of data with respect to the unknown true value. Small Standard Errors imply high accuracy. Standard Error is derived from Standard Deviation; it is equal to Standard Deviation divided by the square root of the number of observations in the set. The greater the number of observations, the more confidence we can have in the Standard Error

## Standard Error

- As the number of observations ( $n$ ) increases, Standard Error ( $\sigma_m$ ) decreases
- As  $n$  approaches infinity,  $\sigma_m$  approaches zero, and the mean approaches truth

$$\sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

## Standard Error at 1 Sigma

- The mean of a set of 1000 observations is 28" and the 1 sigma standard deviation is +/-1.6"
- The 1 sigma standard error = 0.051"
- This means:
  - there is a 68.3% probability that the mean is +/-0.051" of the true value

$$= \frac{1.6}{\sqrt{1000}}$$

## Standard Error at 2 Sigma

- The mean of a set of 1000 observations is 28" and the 1 sigma standard deviation is +/-1.6"
- The 2 sigma standard error = 0.10"
- This means:
  - there is a 95.5% probability that the mean is +/-0.10" of the true value

$$= \frac{1.6}{\sqrt{1000}} \times 2$$

## Standard Error at 3 Sigma

- The mean of a set of 1000 observations is 28" and the 1 sigma standard deviation is +/-1.6"
- The 3 sigma standard error = 0.15"
- This means:
  - there is a 99.7% probability that the mean is +/-0.15" of the true value

$$= \frac{1.6}{\sqrt{1000}} \times 3$$

## Standard Error

- As the number of observations ( $n$ ) increases, Standard Error ( $\sigma_m$ ) decreases
- As  $n$  approaches infinity,  $\sigma_m$  approaches zero, and the mean approaches truth

$$\sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

## Standard Error

- Standard Error can be used to design an observation program...
- knowing  $\sigma_s$ , we can find the number of observations,  $n$  required to achieve a desired accuracy,  $\sigma_m$ , in our measurements

$$\sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

If you know the standard deviation of a measurement system, you can use the standard error equation to compute a value for  $n$ , or the number of observations necessary to arrive at a given accuracy in the result.

## Example:

- Your 2 sigma Standard Deviation ( $\sigma_s$ ) for horizontal angle measurements is +/- 3.2" with a particular instrument...

$$\sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

Even though this instrument and its operator can measure an angle accurately to "only" +/-3.2", better accuracy can be achieved simply by taking more observations.

The question is, how many observations will be needed to get to the desired accuracy level?

## Example:

- Your 2 sigma Standard Deviation ( $\sigma_s$ ) for horizontal angle measurements is +/- 3.2" with a particular instrument...

$$\sigma_m = \frac{3.2}{\sqrt{n}}$$

Here, we will substitute 3.2 for standard deviation in the formula.

## Example:

- ...but you need to obtain mean horizontal angles that are accurate to +/-1" ( $\sigma_m = 1$ )...
- ...how many angle observations must you collect with this instrument to achieve the desired accuracy?

$$\sigma_m = \frac{3.2}{\sqrt{n}}$$

We need angle accurate to 1" from a 3.2" gun and operator...

## Example:

- You need to obtain mean horizontal angles that are accurate to +/-1" ( $\sigma_m = 1$ )...
- How many observations must you collect with this instrument to achieve the desired accuracy?

$$1 = \frac{3.2}{\sqrt{n}}$$

Next we'll substitute 1 for the standard error---this is the accuracy level we want.

## Example:

- Solving for  $n$ ...

$$1 = \frac{3.2}{\sqrt{n}}, \sqrt{n} \times 1 = 3.2, n = 3.2^2, n = 10.6$$

- With this instrument, you should collect at least six sets of angles (= 12 angle measurements) to achieve the desired level of accuracy...

Doing the math, we find that we'll need to take at least 6 sets of direct and reverse angles to achieve the desired accuracy level.

## Example:

- Solving for  $n$ ...

$$1 = \frac{3.2}{\sqrt{n}}, \sqrt{n} \times 1 = 3.2, n = 3.2^2, n = 10.6$$

- ...and you should reject any observation that differs from the mean of the sets by more than  $\pm 3.2$ "

As you collect the data, be sure that you eliminate "outliers" or observations that exceed the standard deviation.

## References

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